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## RESEARCH ARTICLE

# INVERSION AND COMPLEXITY ANALYSIS OF WAVE PARAMETERS APPLIED TO SPECTRAL RECOMPOSITION OF GROUND PENETRATING RADAR (GPR) DATA

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### ABSTRACT

Ground Penetrating Radar (GPR) method is important to perform the structural characterization in near-surface surveys. Obtaining information of frequency and amplitude increases strongly the possibility of performing a better stratigraphical characterization. The spectral recomposition is an efficient method to recover wave parameters information in near-surface seismic data processing. Since many wavelets observed in seismic records are similar to the ones observed in GPR data, it is possible to recompose the GPR signal spectrum using the same proposition. For this reason, we propose to apply, for a GPR signal, an adapted version of the spectral recomposition approach, aiming to obtain, through an inversion procedure, frequency and amplitude information in a more efficient manner. Since the peak frequencies observed in GPR signal are more singular and higher, in comparison to the seismic signal, it is possible to recover frequency and amplitude information in a more accurate manner.

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## INTRODUCTION

Frequency-domain information can provide a better resolution concerning the geological characterization; and knowing each set of frequencies and amplitudes which composes a spectrum can improve the modelling for near-surface analysis (6, 20). Ground Penetrating Radar (GPR) is commonly used in near-surface surveys, and is based on the using of electromagnetic radiation in a high frequency in the order of radio/microwave frequencies (1, 2, 10, 12, 28). Since the range of frequencies and the peak frequencies used in GPR surveys are, respectively, broader and higher, in comparison to seismic, this electromagnetic method is for efficient to be used for near-surface targets; however, due to the similarities of the seismic and GPR methods during some steps of the data processing (5, 9, 11, 14), some techniques used in seismic processing can be adapted for GPR processing. For years, the spectral decomposition technique was studied and applied for stratigraphic characterization in seismic data processing for many cases, for instance, such as, time-frequency analysis,

short-time Fourier transform, and time-domain spectral decomposition (7, 8, 13, 21, 22, 24). Instead of decomposing the frequencies of the spectra by time-domain analysis; Tomasso et al. (27) proposed an approach which is able to recompose the single frequencies into a multi-frequencies model. This method is based on describing the seismic spectrum as a sum of different Ricker wavelet (25) components. The limitation of this method resides in the necessity of picking manually each pair of amplitudes and peak frequencies. However, Cai et al. (4) proposed to estimate the linear part and the nonlinear part of the Ricker wavelet spectrum in an automated manner; with the nonlinear part estimation based on using separable nonlinear Least Squares estimation, which estimates automatically both parts of the spectrum (15). This approach provides an accurate estimation of the amplitudes and peak frequencies of multiple Ricker wavelets. Once the GPR signal commonly behaves as Ricker and/or semi-Gaussian wavelets, the described method of spectral recomposition could be an efficient way to obtain frequency and amplitude information.

This idea is strengthened once it is assumed that the peak frequency of a GPR signal is better defined — presenting a single peak frequency —, and presents higher frequencies, in comparison to the seismic signal. For this reason, the approach we propose is very desirable for GPR signals.

## METHOD

**Relation between GPR and seismic methods:** GPR and the seismic methods present similarities between them, excepting for the use of electromagnetic energy rather than the acoustic energy. For this reason, the interfaces between two layers with different physical properties can appear in different positions in each method, since the observed boundaries are dependent of the electrical properties and not dependent of the mechanical properties (5, 28). As there is a close relation between seismic and GPR methods, and many techniques usually used in seismic can be adapted and applied to be used on GPR data processing (5). An important comparison between both methods is that the acquisition array of the seismic method is usually in function of the source with many receivers in different and periodic distances, providing an event which tends to a hyperbola (21, 22). On the other hand, the GPR presents one source and one receiver with the same offset, which provides a straight event (10, 14). Despite that, there are many GPR acquisition arrays similar to seismic surveys, which requires, during the processing, techniques used in seismic data processing, such as, for instance, NMO correction, Kirchoff migration and stacking (3, 11, 26). For this reason, the technique of spectral reconstruction (27) has potential to be used for GPR processing (32).

**Ricker and semi-Gaussian wavelets:** One of the main differences between GPR and seismic signals are concerning the wavelet. In the seismic signal, the phase, usually, tends to be zero and usually changing this characteristic when it goes through a phase shift due to a critical reflection; and, in this case, the main shape observed is the Ricker wavelet (16, 29) — the second derivative of the Gaussian function. The Ricker wavelet (Ricker, 1953) can be described as

$$\psi(t) = \frac{2}{\sqrt{3\sigma\pi^4}} \left(1 - \left(\frac{t}{\sigma}\right)^2\right) \exp\left(-\frac{t^2}{2\sigma^2}\right) \quad (1)$$

where  $\sigma$  is the wavelength and  $\omega$  is the dominant wavelength. After the critical point, and, therefore, the phase shift, there is more signals with a displaced (between than  $0^\circ$  and  $180^\circ$ ) or reversed (for around  $180^\circ$ ) Ricker wavelet signals. For this reason, the semi-Gaussian wavelet — the first derivative of the Gaussian function — is frequently observed (30). As the modulus of the derivative in the Equation 1 with respect to  $t$  is applied, it results in

$$\psi(t) = \frac{2t}{\sqrt{3\sigma\pi^4}} \exp\left(-\frac{t^2}{2\sigma^2}\right), \quad (2)$$

the semi-Gaussian function.

Applying the modulus of the derivative to Equation 2 results in Equation 3, the Gaussian function.

$$\psi(t) = \frac{2\sqrt{\sigma^3}}{\sqrt{3\pi^4}} \exp\left(-\frac{t^2}{2\sigma^2}\right) \quad (3)$$

Regarding the GPR signal, the Ricker and the semi-Gaussian wavelets appear very frequently — among many other wavelets (17). For this reason, it is necessary to use techniques for spectral analyses that are capable of consider, at least, these two kinds of wavelet (Figure 1).

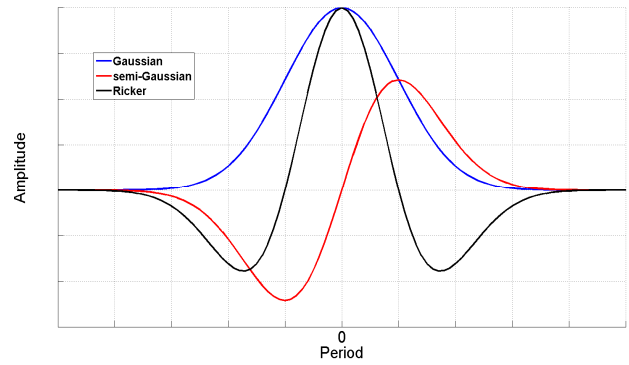


Figure 1. Gaussian, semi-Gaussian, and Ricker wavelets

**Spectral reconstruction:** The spectral decomposition is a technique of time-frequency analysis used to perform stratigraphic characterization with the seismic method. However, to recompose single frequencies into a multi-frequencies model can provide a set of valuable information that can enhance the stratigraphic model (27). The definition of frequency recomposition is described as an estimation of components of a spectrum; however, this definition was proposed to estimate components of seismic spectrum to make forward models (27). This method provided the possibility of reconstructing the seismic spectrum manually picked, and not decomposing it. The limitation of this approach in performing the manually picking of each pair of amplitudes and peak frequencies can be overcome by using the proposition of the nonlinear estimation (15). The estimation of linear and nonlinear parts of the Ricker spectrum performed in an automated manner (4). Cai et al. (4) proposed an automated manner to perform the identification of the frequency spectrum for each trace aiming to recover the information of amplitude and frequency using the spectral reconstruction proposed by Tomasso et al. (27). Tomasso et al. (27) propose to describe the seismic spectrum as a sum of different Ricker components as observed in Equation 1.

$$d(f) \approx \sum_{i=1}^n a_i \psi_1(m_i, f), \quad (4)$$

where  $d(f)$  is the spectrum of a seismic trace,  $f$  is the frequency, and  $a_i$  and  $m_i$  are, respectively, the amplitude and the peak frequency of the  $i$ -th Ricker spectrum component, given by

$$R(f) = a\psi(m, f) = a \frac{f^2}{m^2} \exp\left(-\frac{f^2}{m^2}\right) \quad (5)$$

As the description of the wavelet is known, it is possible to perform an inversion aiming to fit the calculated frequency spectrum to the observed frequency spectrum of the wavelet for each trace.

The spectrum definition proposed by Tomasso et al. (27) can include the characterization of the spectrum of a Ricker and semi-Gaussian wavelet (4, 31, 32). For this reason, the approach is, in this work, proposed to be applied for GPR

The spectrum definition proposed by Tomasso et al. (27) can include the characterization of the spectrum of a Ricker and semi-Gaussian wavelet (4, 31, 32). For this reason, the approach is, in this work, proposed to be applied for GPR signal. An important consideration is that the GPR signal usually presents a peak frequency more well define in comparison to the seismic signal, which presents a set with more peak frequencies. For this reason, it is very interesting to apply this approach for GPR signals, aiming to recover wave parameters more accurately. The GPR signal spectrum can be represented as a sum of Ricker components, in the same way as the seismic spectrum. However, since the GPR signal presents a higher and single peak frequency in comparison to the seismic signal, the proposed approach becomes simpler to be implemented, since the inversion must be performed only for a pair of amplitude and peak frequency.

**Topological analysis of the RFM of the frequency spectrum:** The complexity analysis is important, once the objective function can be described as the quality of a solution in relation to its variables. In this work, the estimation of the wave parameters was performed by minimizing the error with the Least Squares method by approximating the solution. The method is based in reaching the best fit for a set of data by minimizing the sums of the squares of the differences between the estimated values and the observed ones. In this work, the Least Squares method was used to perform the inversion aiming to measure how close the estimated spectrum and observed spectrum are; with the minimum value of the function, there is the optimum value for the solution, once the sum of the squares of the differences between the observed spectrum and the estimated one is the least possible.

This analysis can be spread for any set of combination of parameters. For this, it is necessary to determine an area of to analyse the data. This area determines the maximum and the minimum value of the parameter to be estimated, providing several sets of parameters, in which most of them are not the solutions; however, they show how the error varies until it reaches the optimum set of values. This kind of complexity analysis is known as Residual Function Maps (RFM); it allows studying the topology of the function and how the error varies by mapping the objective function (18, 19). This method is very appropriate to analyse problems with two variables, and it is important to understand the complexity of the objective function, which provides information, such as, about the best kind of optimization algorithm to be used, or about the behaviour of the parameters for many different solutions (34, 36, 37). In this work, the RFM was constructed with two dimensions related to the wave parameters — one dimension representing the amplitude, and the other one representing the frequencies — while the third dimension in the hyperplane represents the value of the objective function, or, in other words, the error between the calculated and the observed values.

Three peak frequencies were adopted to perform the tests (Figure 2); all of them are frequencies which are commonly used for near-surface investigation: 900 MHz, used for targets shallower than around 1 meter depth; 600 MHz, used for targets around 1.5 meters depth; and 300 MHz, which is used for targets until around 2.5 meters depth. In Figure 3, the RFM are placed to show the variation of the complexity of the topology of the objective function for each of the three frequencies tested in this work. In Figure 3A, it is possible to

observe that, for 900 MHz, there is a homogeneity concerning the variation of both parameters to be recovered (amplitude and frequency), which allows, proportionally, a similar accuracy to recover both parameters. For 600 MHz (Figure 3B), it is possible to observe that the topology is less homogeneous between frequency and amplitude. In this case, the resolution to recover the frequency is increased, once the limits for the optimization does not exceed the 1000 MHz; however, with less initial information, and, therefore, with the necessity of a larger area to analyse, many optimization algorithms can be trapped in solutions related to higher frequencies, or, at least, take a higher processing time to find the global minimum region.

This kind of behaviour increases for 300 MHz (Figure 3C), and it is possible to observe that the resolution increases even more; however, the boundaries for the initial starting optimization points should be set in a region no higher than 500 MHz, since, above that value, the variation starts to appear almost only for the amplitude (almost parallel to the frequency axis). The RFMs with a broader frequency axis (Figure 3D, Figure 3E, and Figure 3F) show the importance of setting a limited starting region to set the initial points to perform the optimization, since the significantly higher values of frequencies, in comparison to the peak frequency, bring the same behaviour of making the contour lines getting almost parallel to the frequency axis, which brings difficulty to recover peak frequency information.

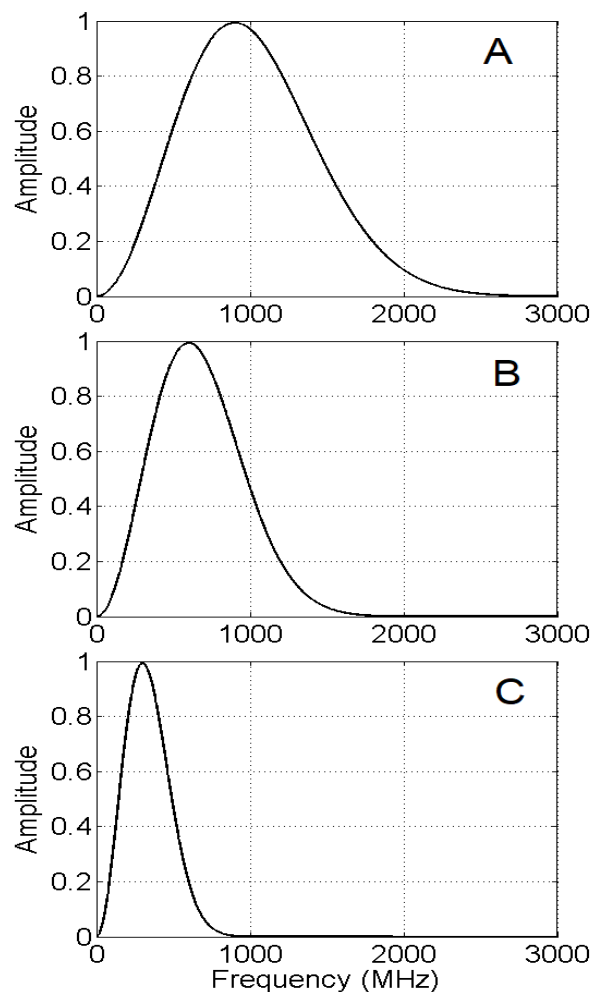
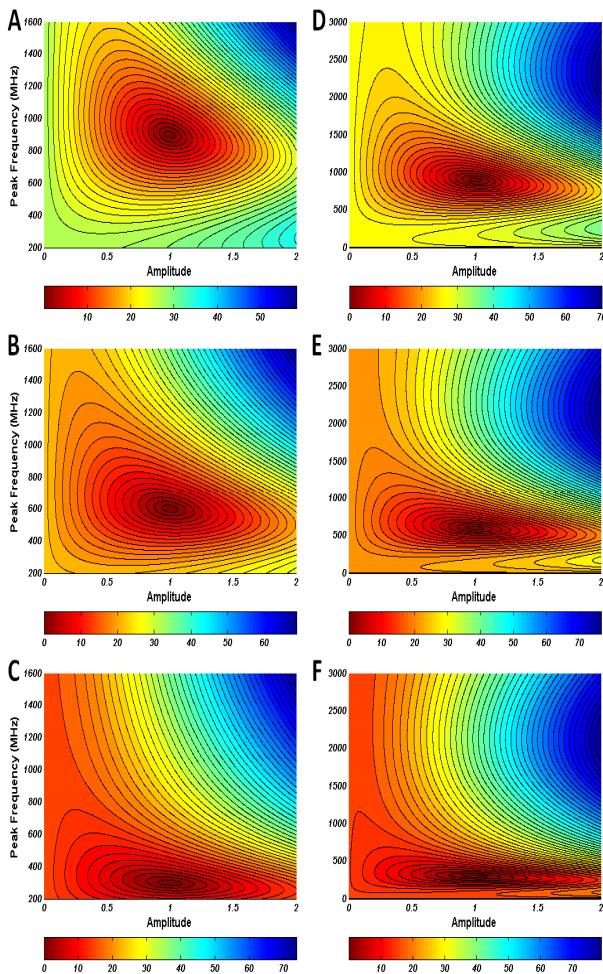


Figure 2. Ricker wavelet spectrum for (A) 900 MHz, (B) 600 MHz, and (C) 300 MHz



**Figure 3. Residual function maps to demonstrate the complexity of the topology of the objective function for (A) 900 MHz, (B) 600 MHz, and (C) 300 MHz. The RFM with a broader peak frequency axis for (D) 900 MHz, (E) 600 MHz, and (F) 300 MHz.**

**Inversion of frequencies and amplitudes:** To obtain the information of amplitude and frequency of the GPR spectrum, an inversion procedure according an optimization criterion was proposed. The curve calculated with the Ricker wavelet spectrum was fit to the observed spectrum, using the adapted simplex optimization algorithm proposed by Nelder and Mead (23), and the Least Squares as minimization norm. This local search optimization algorithm can be used, since the main area of the analysed topology presented only a global minimum region (33, 35). Multi-start procedure was applied, since the selected algorithm is for local search optimization, aiming to reach the global minimum region. With this procedure, the global minimum can be found even whether there are local minimum regions not found during the topological analysis of the RFM. The procedure is based on perform several inversions, with each one starting from an initial random point.

Two important factors regarding the amplitude recovering were studied in previous works (4, 21, 32): 1-the higher is the amplitude the more accurate is the procedure to recover the amplitude information during the inversion; 2-the higher is the frequency, in comparison with other frequencies of the spectrum, the more accurate is to recover the amplitude information during the inversion. For this reason, it is very interesting to understand, in a better way, how the magnitude of the frequency influences the information recovering during

the inversion, once the GPR uses a much broader frequency variation than the one used in seismic methods. Since the difficulty to recover the amplitude is well known, it was normalized to have a better comparison between the accuracy to recover the information for different frequencies. As it is possible to observe in Table 1, for each frequency tested (900, 600 and 300 MHz), the resolution in the direct modeling is lower for higher frequencies, due to its softer variation and similar along both axes. However, even though a lower frequency presenting a better resolution, it showed the necessity of limit the boundaries in a smaller space around the solution, which is a problem when there is only little initial information. Regarding the residual error with no noise, higher frequencies presented a lower error, which increased exponentially when the frequency got lower. Concerning the residual error with no noise, it was possible to observe that the same pattern of the error, which got higher while the frequency decreased; however, the variation was much softer, in comparison to the residual error variation with no noise.

**Table 1. Average Frequency Resolution, Average Residual Error, and Average Residual Error with noise for each frequency tested**

Frequency (MHz)	Frequency Resolution (MHz)	Residual Error (%)	Residual Error with Noise (%)
900	52	0.0102	3.11
600	41	0.0228	7.99
300	32	0.1009	12.35

## CONCLUSION

The approach we propose to be adapted to GPR signal presented to work in an efficient manner, and it shows to be an effective manner to recover frequency and amplitude information, which is essential to perform a better stratigraphical characterization in near-surface investigations using GPR method. With a lower error, the higher frequencies are better recovered, and therefore, allows a better stratigraphical characterization for shallower targets.

This does not prevent to apply the approach for lower frequencies, and, consequently, for deeper targets; however, the application for lower frequencies generates more problems concerning the ambiguity of the information obtained, which demands more a priori information to restrict the limits in order to set the initial parameters to perform the inversion procedure. Even though the approach tends to be used for shallower investigation — since it presents a higher accuracy in obtaining higher frequencies —, the accuracy to obtain the frequency and amplitude information is, in many cases, considerably acceptable for lower frequencies, if it is not demanded to be highly accurate.

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